Probability

P(AuB) = P(A) + P(B) - P(AnB)

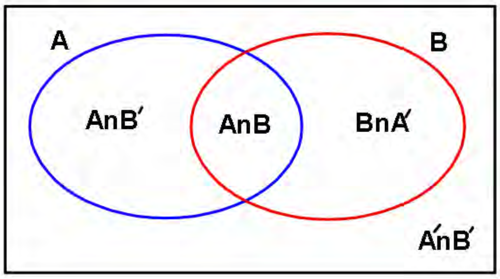
this can easily be rearranged to give probability that BOTH events occur; simply swap the n and u

P(AnB) = P(A) + P(B) - P(AuB)

$$P(A|B) = \frac{P(AnB)}{P(B)}$$

P(A|B) → n in the numerator

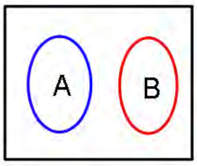
P(A|B) → divide by this



Mutually exclusive

P(AnB) = 0 No overlapping

P(AuB) = P(A) + P(B)



Independent if

P(AnB) = P(A) x P(B)

Discrete Random Variables

Probability function is a rule for finding probabilities from the random variable

P(X=x) =	kx	if x = 1, 2 and 3
	(k-3)x	if x = 4 and 5
	0	otherwise

A probability distribution lists the random variables and their associated probabilities

x	1	2	3	4	5
P(X=x)	k	2k	3k	k	2k

$$\sum P(X = x) = 1$$

So k + 2k + 3k + k + 2k = 1 9k = 1 $k = \frac{1}{9}$

x	1	2	3	4	5
P(X=x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

The cumulative probability distribution is denoted with an F
(the final cumulative probability must be equivalent to 1.

x	1	2	3	4	5
F(X=x)	$\frac{1}{9}$	$\frac{3}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{9}{9}$

$$E(X) = \sum x\, P(X = x)$$ $E(X)$ is the MEAN the EXPECTED value

x	1	2	3	4	5
P(X=x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

$$E(X) = 1 \times \frac{1}{9} + 2 \times \frac{2}{9} + 3 \times \frac{3}{9} + 4 \times \frac{1}{9} + 5 \times \frac{2}{9} = \frac{29}{9}$$

$$E(x^2) = \sum x^2\, P(X = x)$$

x^2	1	4	9	16	25
P(X=x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

$$E(X^2) = 1 \times \frac{1}{9} + 4 \times \frac{2}{9} + 9 \times \frac{3}{9} + 16 \times \frac{1}{9} + 25 \times \frac{2}{9} = \frac{102}{9}$$

$V(X) = E(X^2) - E(X)^2$

$$V(X) = \frac{102}{9} - \left(\frac{29}{9}\right)^2 = \frac{77}{81}$$

$$sd = \sqrt{\frac{77}{81}} = 0.975\ (3sf)$$

E(aX+b) = a x E(X) + b

V(aX+b) = a^2 x V(X)

Remember + or - does not affect a spread

Using the code Y = 5 - 3X Find the mean and standard deviation of Y

$$E(Y) = E(5 - 3X) = 5 - 3\,E(X) = 5 - 3 \times \frac{29}{9} = -\frac{14}{3}$$

$$V(Y) = V(5 - 3X) = (-3)^2\, V(X) = 9 \times \frac{77}{81} = \frac{77}{9}$$

$$sd_y = \sqrt{\frac{77}{9}} = 2.92\ (3sf)$$

So, the mean of y is -4.67 and its standard deviation is 2.92

Discrete Uniform Distributions

Each value must be equally likely (i.e. rolling a fair dice)

$$P(X = x) = \frac{1}{n}$$

$$E(X) = \frac{n+1}{2}$$

$$V(X) = \frac{(n+1)(n-1)}{12}$$

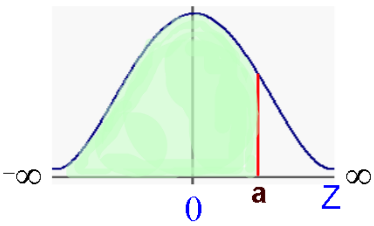
The Standard Normal Distribution, Z

This is a special normal distribution which has a mean of zero and standard deviation of 1 this is denoted by Z i.e. $Z \sim N(0,1^2)$

The Total area under the curve of the standard normal distribution is 1

Any normal distribution X can be transformed into Z by **subtracting its mean** and **dividing by its standard deviation**

$$Z = \frac{X - \mu}{\sigma}$$

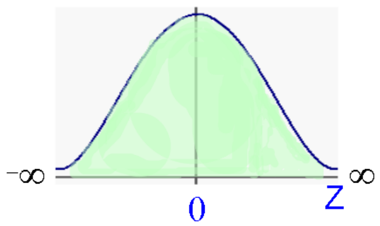


The shaded area represents $P(Z \leq A)$

The area under the standard normal distribution curve between $-\infty$ and a has been tabulated for all postive values of a in Statistics tables it is denoted by $\phi(a)$. To calculate the area for negative values of a we use the fact it is symmetrical about the y axis and has a total area of 1

TIP - READ ϕ(a) AS THE AREA TO THE LEFT OF a

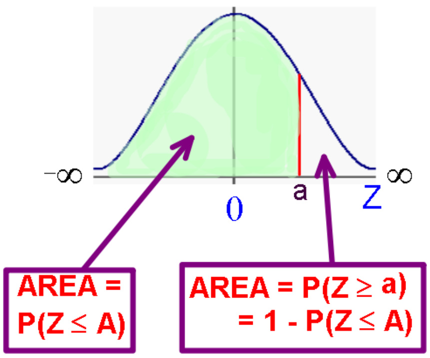
STANDARD NORMAL DISTRIBUTION Z KEY POINTS



PERFECTLY SYMMETRICAL

TOTAL AREA = 1   0·5 EACH SIDE OF VERTICAL AXIS

SHADED AREA REPRESENTS PROBABILTY



AREA = $P(Z \leq a)$ DENOTED BY $\phi(a)$

STATS TABLES ONLY GIVE AREA FOR POSITIVE VALUES OF a   i.e SMALLEST AREA GIVEN IS FOR $a = 0$ FOR WHICH $\phi(a) = 0{\cdot}5$

TO FIND $P(Z \leq a)$ FOR NEGATIVE VALUES OF a USE SYMMETRY PROPERTIES

TO FIND $P(Z \geq a)$ USE SYMMETRY PROPERTIES

KEY FEATURES OF NORMALLY DISTRIBUTED DATA

ONLY USED WITH CONTINUOUS DATA

PERFECTLY SYMMETRICAL ABOUT THE MEAN

HORIZONTAL AXIS ASYMPTOTIC TO CURVE

DISTRIBUTION IS BELL SHAPED

68.3% OF DATA LIES WITHIN 1 STANDARD DEVIATION OF THE MEAN

95% OF DATA LIES WITHIN 2 STANDARD DEVIATIONS OF THE MEAN

99% OF DATA LIES WITHIN 3 STANDARD DEVIATIONS OF THE MEAN

Solving Normal Distribution Problems - tips

Create a probability equation(s) from the context of the question

Turn into the normal distribution into the standard normal distribution Z
(subtract mean then divide by standard deviation)

Simplify value

If possible manipulate the probability so that z is positive
i.e. $P(Z < -1)$ then change to $P(Z > 1)$

To change a negative value of z to a positive a z simply reverse the inequality.

If possible manipulate the probability so that the inequality is <
i.e. $P(Z > k) = 0.2$ then change to $P(z < k) = 1 - 0.2 = 0.8$

turn > to < by subtracting from 1.

Turn into ϕ

If necessary manipulate ϕ so that probability is above 0.5
i.e if $\phi(k - \mu) = 0.2$ then change to $\phi(\mu - k) = 1 - 0.2 = 0.8$
switch *subtract from 1*

Use the Normal distribution tables
if you know a $\phi(0.8) = ?$ look in the left hand column for the given value
if you know the probability $\phi(?) = 0.8$ look in the right hand column for the given value

Solve it

DRAW DIAGRAMS TO HELP YOU SOLVE THE PROBLEM

PROB IT
ZED IT
PHI IT
TABLE IT
SOLVE IT

Use Percentage points table if you know the probability > a and it is a nice value